Місця першого класу знаходяться на другому поверсі, вони найбільш комфортні й дорожче коштують.

Висновок

Розробка та впровадження швидкісного руху по всій території України дозволить залізничному транспорту підвищити відсоток пасажирських залізничних перевезень. Впровадження швидкісного руху на ділянках Київ - Полтава - Харків; Київ - Донецьк; Київ - Дніпропетровськ; Київ - Одеса; Київ - Львів; Харків - Запоріжжя - Сімферополь дозволить пасажирам скоротити час перебування в дорозі, в наслідок чого збільшиться відсоток користувачів залізничними послугами не тільки на час проведення ЄВРО-2012.

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ДОСТАТНЯ УМОВА СКІНЧЕННОСТІ ТОЧКОВОГО СПЕКТРУ ТРАНСПОРТНОГО ОПЕРАТОРА

Встановлено, що транспортний оператор за деяких умов може мати тільки скінченний точковий спектр.

Ключові слова: оператор, перетворення Фур'є, аналітичне продовження.

Установлено, что транспортный оператор при некоторых условиях может иметь только конечный точечний спектр.

Ключевые слова: оператор, превращение Фур'е, аналитическое продолжение.

Abstract: It is proved that transport operator under certain conditions can have finite point spectrum only.

Keywords: operator, spectrum, Fourier transformation, analytic extension.

1.Statement of the problem

We consider partial case of so-called "equation of transmission". There is much literature concerning (during many years) different problems in this direction. One of such problems, namely the problem of neutron transport, leads to the operator

$$Lf(x,\mu) = -i\mu\frac{\partial f}{\partial x}(x,\mu) + c(x)\int_{-1}^{1} f(x,\mu')d\mu' \qquad (1)$$

in the space $L^2(D)$, where $D = R\times[-1,1]$. In [1] in the case

$$c(x)=\begin{cases} c, & |x|<a \\ 0, & |x|>a, c=\text{const} \end{cases}$$

it was obtained that continuous spectrum coincides with real axis R and that the set of eingen-values is finite. In [2] in the case $c\in L^{\infty}(R), \operatorname{supp} c\subset[-a,a], c(x)\geq 0$ well-known functional model is applied.

In [3] the authors use Friedrichs' model to study the operator L. In the case of exponentialy decreasing potential the sufficient condition of finiteness of point spectrum was obtained. The methods of this work were used in [4] in more general case of the operator

$$Lf(x,\mu)=-i\mu\frac{\partial f}{\partial x}(x,\mu)+a(x)\int_{-1}^{1}b(\mu')f(x,\mu')d\mu'. \tag{2}$$

As it was proved in [4] the value $\zeta=0$ only can be the point of accumulation of point spectrum of the operator L if the following conditions hold:

a)the function $a(x)$ is locally integrable and satisfies the estimate

$$|a(x)|\leq Me^{-\varepsilon|x|}, x\in R, \tag{3}$$

where $\varepsilon>0, M>0$ are some constants.

b)the function $b(\mu), \mu\in(-1,1)$ admits analytic extension $b(z)$ into the circle $|z|<1+\varepsilon$.

Note, that the resolvent admits analytic extension over $(-\infty,0)$ and $(0,\infty)$ (see Theorem 1, Theorem 2 of [4]).

The aim of our article is to give sufficient condition on the functions $a(x)$, $b(\mu)$ such that the value $\zeta=0$ is not the point of accumulation of point spectrum. Then the point spectrum of the operator L is finite set. Like [3-4] we use unitary equivalence of the operator L to the operator of Friedrichs' model.

2.Friedrichs' model

Here we give some notations and results from [4].

Let H be Hilbert space of the functions on two variables $\varphi(s,\mu), (s,\mu)\in D=R\times[-1;1]$ with norm

$$\|\varphi\|_H^2=\int_R\int_{-1}^{1}|\varphi(s,\mu)|^2\frac{1}{|\mu|}dsd\mu$$

and let $G=L^2(R)$. We denote by $(\cdot,\cdot), (\cdot,\cdot)_H$ scalar product in the spaces G and H respectively. We denote by $S: H\to H$ the operator of multiplication by independent variable $(S\varphi)(\tau,\mu)\equiv\tau\varphi(\tau,\mu), \tau\in R$ with maximal domain of definition. Using Fourier transformation it was proved in [4] that the operator $L: L^2(D)\to L^2(D)$ is unitary equivalent to the operator $T=S+A^*B: H\to H$ with bounded operators $A^*: G\to H$, $B: H\to G$ under the form

$$A^*c(s,\mu)=\frac{1}{2\pi}\int_R a_1(y)c(y)e^{-iy\frac{s}{\mu}}dy, \tag{4}$$

and

$$B\varphi(x)=a_2(x)\int_R e^{ix\tau}\left(\int_{-1}^{1}b(\mu')\varphi(\tau\mu',\mu')d\mu'\right)d\tau. \tag{5}$$

We use the traditional form of perturbation A^*B, that's why we don't need the operator $A: H \to G$ itself. The representations (4)-(5) contain the factors $a_{1,2}(x)$ of arbitrary factorization such that

$$a(x) = \overline{a_1(x)} a_2(x), \ |a_1(x)| = |a_2(x)|.$$

The relation between the resolvents $T_\zeta = (T-\zeta)^{-1}$ and $S_\zeta = (S-\zeta)^{-1}$ of the operators T and S is $T_\zeta = S_\zeta - S_\zeta A^* K(\zeta)^{-1} B S_\zeta$, where $K(\zeta) = 1 + BS_\zeta A^*$.

3. Estimate of the operator $K(\zeta)-1$

As it was proved in [4(Lemma 3.2, Lemma 3.3)] that

$$((K(\zeta)-1)g)(x) = \int_R k(x,y,\zeta)g(y)dy, \ k(x,y,\zeta) = \frac{1}{2\pi} a_2(x)a_1(y)I(x-y,\zeta), \text{ where}$$

$$I(u,\zeta) = \int_0^\infty \frac{1}{t-\zeta} f_{-s(u)}(t|u|)dt - \int_0^\infty \frac{1}{t+\zeta} f_{s(u)}(t|u|)dt, \ s(u) = sign(u) \text{ and}$$

$$f_\omega(\theta) = \int_\theta^\infty \frac{1}{y}\left[b\left(\frac{\theta}{y}\right)e^{-i\omega y} + b\left(-\frac{\theta}{y}\right)e^{i\omega y}\right] dy, \omega = \pm 1.$$

Concerning the function $f_\omega(\theta)$ it was obtained the estimate $|f_\omega(\theta)| < a_1|\ln\theta| + a_2, 0<\theta<1$ and $|f_\omega(\theta)| < \frac{a}{\theta}, \theta > 1$, where a_1, a_2, a are some constants with respect to the variable θ. Easy to see that $a_1, a_2, a \le C\|b\|_{C^1}$, where $C = const$ doesn't depend on the function $b(\bullet)$.

Let $\Omega_\pm(\delta) = \{\zeta : |\zeta| < \delta, \pm \operatorname{Im}\zeta > 0\}$, $\|a\|_\delta^2 \equiv \int_R |a(x)|^2 e^{2\delta|x|}dx$,

$$M(\delta) = C_1\left(\int_R\int_R \left[|y-x|^{-\frac{4}{p}} + (y-x)^4 \right] \exp[-2\delta(|x|+|y|)]dxdy \right)^{\frac{1}{2}} \qquad (6)$$

for some $p > 4$, where $C_1 = const$ doesn't depend on the functions $a(\bullet), b(\bullet)$.

3.1.Theorem. For every $\delta < \varepsilon$ (see(3)) the operator $K(\zeta)$ has form

$$K(\zeta) - 1 = \pm\pi i b(0)\ln\zeta(\bullet, a_1)a_2 + Q(\zeta), \ \zeta \in \Omega_\pm(\delta), \qquad (7)$$

where $\ln\zeta$ denotes the branch of logarithmic function which is continuous in the domain $\zeta \notin [0,\infty)$ and such that $\ln(-1) = \pi$. The operator $Q(\zeta): L^2(R) \to L^2(R)$ is compact and bounded uniformly with respect to ζ, namely:

$$\|Q(\zeta)\| \le M(\delta)\|b\|_{C^1}\|a\|_\delta, \zeta \in \Omega_\pm(\delta), \qquad (8)$$

where constant $M(\delta)$ is defined by (6). The proof of theorem 3.1 is similar to one of Lemma 4.2 from [3]. Also Lemma 3 from [4] is important.

4.Finiteness of the spectrum

We introduce scalar function $\gamma(\zeta)$ and operator function $\Gamma(\zeta)$ by the relations:

$$\gamma(\zeta) = \pm\pi i b(0)\ln\zeta, \ \Gamma(\zeta)c = c + \gamma(\zeta)(c,a_1)a_2, \zeta \in \Omega_\pm(\delta). \qquad (9)$$

4.1.Lemma. Suppose that $b(0)(a_2,a_1) \ne 0$. Let's choose the value $\delta_1 < 1, 0 < \delta_1 < \delta$ such that (see(8))

$$|\gamma(\zeta)(a_2,a_1)| \equiv \pi|\ln\zeta| \cdot |b(0)| \cdot |(a_2,a_1)| \ge 2, 0 < |\zeta| \le \delta_1 \qquad (10)$$

then the inverse operator $\Gamma(\zeta)^{-1}$ exists for the coresponding values ζ and its norm has the estimate:

$$\left\|\Gamma(\zeta)^{-1}\right\| \le 3\frac{\|a_2\|\cdot\|a_1\|}{|(a_2,a_1)|},\ 0<|\zeta|<\delta_1. \qquad (11)$$

Proof. Let's consider the equation $\Gamma(\zeta)c=d$ or (see(9))

$$c+\gamma(\zeta)(c,a_1)a_2=d. \qquad (12)$$

Multiplying by a_1 we get $(c,a_1)[1+\gamma(\zeta)(a_2,a_1)]=(d,a_1)$. Substitution of the value (c,a_1) in (12) gives

$$c=\Gamma(\zeta)^{-1}d=d-\frac{\gamma(\zeta)}{1+\gamma(\zeta)(a_2,a_1)}(d,a_1)a_2,\ 0<|\zeta|<\delta_1. \qquad (13)$$

We denote $z=\gamma(\zeta)(a_2,a_1)$, then according to the condition (10), we have $|z|>2$, so

$$\left|\frac{\gamma(\zeta)}{1+\gamma(\zeta)(a_2,a_1)}\right|=\frac{1}{|(a_2,a_1)|}\left|\frac{z}{z+1}\right|\le\frac{1}{|(a_2,a_1)|}\cdot\frac{|z|}{\left||z|-1\right|}<\frac{2}{|(a_2,a_1)|}.$$

Therefore,the estimate of the norm of the operator (13) is following:

$$\left\|\Gamma(\zeta)^{-1}\right\|\le 1+\frac{2}{|(a_2,a_1)|}\|a_2\|\cdot\|a_1\|=\frac{|(a_2,a_1)|+2\|a_2\|\cdot\|a_1\|}{|(a_2,a_1)|}\le 3\frac{\|a_2\|\cdot\|a_1\|}{|(a_2,a_1)|},$$

what proves the Lemma.

4.2.Theorem. Let the operator L be given by the expression (2). Suppose that for some value $\varepsilon>0$ the function $a(x)$ satisfies the condition (3) and that the function $b(\mu),\mu\in[-1,1]$ admits analytic extension into the circle $|z|<1+\varepsilon$.

1) Let $b(0)=0$. Let's fix $\delta\in(0,\varepsilon)$.

Then operator L under the condition (see(6))

$$M(\delta)\|b\|_{C^1}\cdot\|a\|_\delta<1 \qquad (14)$$

has not point spectrum in the domain $0<|\zeta|<\delta$;

2) Let $b(0)\ne 0, \int_R a(x)dx\ne 0$. Let's fix $\delta\in(0,\varepsilon)$ and define $\delta_1\le\delta$ according to the condition (10). Then operator L under the condition

$$\|b\|_{C^1}\cdot\|a\|_\delta<M_0(\delta)\left|\int_R a(x)dx\right|/\int_R|a(x)|dx, \qquad (15)$$

where $M_0(\delta)=1/(3M(\delta))$ has not point spectrum in the domain $0<|\zeta|<\delta_1$.

3) Let $\int_R a(x)dx\ne 0$. Let's fix $\delta\in(0,\varepsilon)$.

Then there exist numbers $k_1>0$ and $\delta_1>0$ such that the operator L, formed with the pair of functions $b(\mu)$, $ka(x)$, where $k\in C,|k|<k_1$ has not point spectrum in the domain $0<|\zeta|<\delta_1$.

Proof. It is sufficient to prove the statement of the theorem for the spectrum of the operator T instead of operator L (as T is unitary equivalent to L). Due to [4] it remains to prove that $\zeta=0$ is not the point of accumulation of point spectrum of the operator T. Due to the relation $T_\zeta=S_\zeta-S_\zeta A^*K(\zeta)^{-1}BS_\zeta$ we must study the operator $K(\zeta)^{-1}$. So, let us consider inverse operator for $K(\zeta):G\to G$, or the equation

$$K(\zeta)c=d,\ \zeta\in\Omega_\pm(\delta).$$

1) Let $b(0)=0$. According to (7)-(8) for arbitrary $\delta<\varepsilon$ we have

$$\|K(\zeta)-1\|=\|Q(\zeta)\|\le M(\delta)\|b\|_{C^1}\|a\|_\delta,\ \zeta\in\Omega_\pm(\delta). \qquad (16)$$

According to (14) we have $\|Q(\zeta)\|<1$, then for fixed value $\zeta\in\Omega_{\pm}(\delta)$ inverse operator $K(\zeta)^{-1}$ exists and it is uniformly bounded operator in $\Omega_{\pm}(\delta)$. So, the domains $\Omega_{\pm}(\delta)$ do not contain point spectrum. The operator $T_{\zeta}=S_{\zeta}-S_{\zeta}A^{*}K(\zeta)^{-1}BS_{\zeta}$, where A^{*},B are bounded operators, is bounded for $\zeta\in\Omega_{\pm}(\delta)$. For fixed value $\sigma\in(-\delta,0)\cup(0,\delta)$ limit values of bilinear forms $(T_{\sigma}\varphi,\psi)_{\pm}=\lim_{\tau\to\pm0}(T_{\sigma+i\tau}\varphi,\psi)$ are finite for smooth elements φ,ψ. Really, as it was proved in [4] the operator $K(\zeta)$ admits analytic extension over $(-\infty;0),(0;\infty)$. So, the limit values $Q_{\pm}(\sigma)$ (defined analogically $(T_{\sigma}\varphi,\psi)_{\pm}$) satisfy the inequality $\|Q_{\pm}(\sigma)\|\le M(\delta)\|b\|_{C^1}\|a\|_{\delta}$ like (16). Therefore the circle $0<|\zeta|<\delta$ does not contain the real eigen values from point spectrum of the operator T.

The statement 1) is proved.

2) Let $b(0)\neq 0,\ \int_R a(x)dx\neq 0$. Taking into accont the proof of 1) it's sufficient to prove that the equation $K(\zeta)c=0$ has zero solution only. From the equation $\Gamma(\zeta)c+Q(\zeta)c=0$ (see (7),(11)) we obtain $c+\Gamma(\zeta)^{-1}Q(\zeta)c=0$. We remark that according to factorization $a(x)=\overline{a_1(x)}a_2(x)$ we have $(a_2,a_1)=\int_R a_2(x)\overline{a_1(x)}dx=\int_R a(x)dx.$ As

$$b(0)(a_2,a_1)=b(0)\cdot\int_R a(x)dx\neq 0,$$

then the condition of Lemma 4.1 holds. According to (11) and (8) when $\delta_1\le\delta$ we have

$$\sup_{|\zeta|<\delta_1}\left\|\Gamma(\zeta)^{-1}Q(\zeta)\right\|\le\sup_{|\zeta|<\delta_1}\left\|\Gamma(\zeta)^{-1}\right\|\cdot\sup_{|\zeta|<\delta_1}\|Q(\zeta)\|\le$$

$$\le 3\frac{\|a_2\|\cdot\|a_1\|}{|(a_2,a_1)|}\cdot\sup_{|\zeta|<\delta}\|Q(\zeta)\|\le 3M(\delta)\frac{\|a_2\|\cdot\|a_1\|}{|(a_2,a_1)|}\|b\|_{C^1}\cdot\|a\|_{\delta}.$$

We recall that the factorization $a(x)=\overline{a_1(x)}a_2(x)$ is such that $|a_1(x)|=|a_2(x)|$ then

$$\|a_1\|\cdot\|a_2\|=\left(\int_R|a_1(x)|^2dx\right)^{\frac{1}{2}}\left(\int_R|a_2(y)|^2dy\right)^{\frac{1}{2}}=\int_R|a_1(x)|^2dx=\int_R|a(x)|dx.$$

So, due to the condition (15) we have $\sup_{|\zeta|<\delta_1}\left\|\Gamma(\zeta)^{-1}Q(\zeta)\right\|<1,$ what proves 2).

3) Suppose the inequalities (14) or (15) do not hold for the pair $b(\mu),a(x)$. Then it is evident that they hold for pair $b(\mu),ka(x)$ for sufficient small values k.

Theorem is proved.

Corollary. Operator L (see(2)) under the conditions of Theorem 4.1 can have finite point spectrum only.

Proof. According to [4] the unique point of accumulation of point spectrum of the operator L may be $\zeta=0$ only. But as it follows from the Theorem 4.2 it is impossible, what proves Corollary.

Remark. One can work without Friedrich's model,but with the operator L itself as integro - differential operator. Let us give examples of corresponding factorization.

We have $L=L_0+A^{*}B,$ where

$$L_0 f(x,\mu) = \frac{1}{i}\mu\frac{\partial f(x,\mu)}{\partial x} \quad and \quad A^* B f(x,\mu) = a(x)\int_{-1}^{1} b(\mu') f(x,\mu') d\mu'.$$

Let's choose, for example, the next factorization: $A, B : H \to G,$ where $G = L^2(R)$ and

$$Bf(x) = \int_{-1}^{1} b(\mu') f(x,\mu') d\mu', \; A^* c(x) = a(x)c(x).$$

As

$$L_{0,\zeta} g(x,\mu) \equiv (L_0 - \zeta)^{-1} g(x,\mu) = \begin{cases} -\frac{i}{\mu} e^{\frac{i\zeta}{\mu}x} \int_x^\infty e^{\frac{i\zeta t}{\mu}} g(t,\mu) dt, & \mu > 0 \\ \frac{i}{\mu} e^{\frac{i\zeta}{\mu}x} \int_{-\infty}^{x} e^{\frac{i\zeta t}{\mu}} g(t,\mu) dt, & \mu < 0, \end{cases}$$

then

$$BL_{0,\zeta} A^* c(x) = \left(\int_{-1}^{0} + \int_{0}^{1} \right) b(\mu') f(x,\mu') d\mu' =$$

$$= -i \int_{-1}^{0} \frac{b(\mu')}{|\mu'|} \int_{-\infty}^{x} e^{\frac{i\zeta(x-t)}{\mu}} a(t)c(t) dt d\mu' - i \int_{0}^{1} \frac{b(\mu')}{|\mu'|} \int_{x}^{\infty} e^{\frac{i\zeta(x-t)}{\mu}} a(t)c(t) dt d\mu'.$$

Other factorizations also give complicated expressions for the operator $1 + BL_{0,\zeta} A^*$.

On the other side, we indicate for example the work [5] (see its references too), where for transport operator in bounded domain in R^N compactness directly for the operators like $L_{0,\zeta} V$ and $VL_{0,\zeta}$, $V = A^* B$ was proved with out any Friedrichs' model.

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ВНЕДРЕНИЕ BI РЕШЕНИЙ В CRM СИСТЕМЫ

Стаття присвячена «Бізнес-аналізу» і його значенням в системі управління взаємовідносинами з клієнтами. Описуються підхід в реалізації та специфіка концепції бізнес-аналітичних рішень для системи управління взаємовідносинами з клієнтами у важкій промисловості.
Ключові слова: бізнес-аналіз, системи управління взаємовідносинами з клієнтами

Статья посвящена «Бизнес-анализу» и его значению в системе управления взаимоотношениями с клиентами. Описываются подход в реализации и специфика концепции бизнес-